Day #: 1 **# of minutes:** 62 minutes	**Brief Agenda:** ● Weekly agenda review ● Rationals introduction ● Exploring Rationals using Desmos ● Rationals Practice via Khan Academy

Standard from your discipline:

- **A-APR.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. (Concept)
- **F-IF.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, showing end behavior. (Procedure)
- **F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Application)

Additional standard(s) from another discipline:

- None

Central concept/idea/performance: The central focus of this unit is to introduce rational functions and their properties, like asymptotes and end behavior, and build upon students previous knowledge of polynomials to solve real-world ratio-based problems. Standards A-APR.7 will direct students' conceptual understanding, while F-IF.7d and F-IF.4 will direct student learning of procedure and application, respectively.

Learning Objective(s): Students will be able to identify a rational function when written as an equation as well as when presented graphically, and begin to identify properties of rational functions, like end behavior and vertical asymptotes.

Into *(11:49 - 11:59 am - 10 minutes)*

Activating Prior Knowledge and Making Students Aware of the Central Focus	**Language and Literacy Supports**
● The lesson will start with a review of the weekly agenda, highlighting that this week's central focus will be on rational functions, and that this week will build upon students' knowledge of polynomial functions that they gained the week prior. ● I will briefly introduce rational functions via the projector when communicating this unit's central focus, but only enough to pique students' interest. ● Students will then be directed to move to the exploration-focussed Through activity, described below, while I individually check-in	● ***Interpretive language (i.e., making meaning of texts):*** Students will interpret the brief introduction on rational functions and start building their own mental model. ● ***Collaborative language (i.e., making meaning with others):*** Students will have the opportunity to discuss last week's assessment with both me and their peers, solving any issues together.

with students about the results of their polynomials assessment from last week.	• ***Productive language (i.e., communicating meaning with texts):*** Students may use productive language when describing issues with last week's assessment.
Eliciting Student Ideas and Lived Experiences • Typically, the whole-class Into activity for the first lesson of the week is very brief. Where possible, I'll ask students to identify ratios in their own life and how understanding those ratios helped them in their goals.	**Talk and Translanguaging Moves** • Relate the word "rational" to its Latin root "rationalis", meaning "reasonable" or "logical", and describe how the Greeks specifically chose this word because of their mathematical views of that time period. • Students will be purposely grouped to have ELLs with bilinguals to aid in communication. Students can discuss with a partner in their native language. • When reviewing assessment results with students, I'll ask them to explain their thinking before showing them the correct answer.
Through *(11:59 - 12:41 - 42 minutes)*	
Engaging in Central Learning Activities and Texts • The Through activity for this lesson is structured exploration using Desmos and Khan Academy. • Students start with a Desmos task that introduces rational functions in an interactive graphical environment that treats manipulating graphs of rational functions like a game. The activity is called "Marbleslides: Rationals", and the goal is to manipulate and translate specific rational functions in such a way where balls rolling down the path of the graph hit stars placed throughout the cartesian plane. • Students that finish the Desmos activity can move on to practice questions in Khan Academy where they can dive more deeply into rational functions and their properties.	**Language and Literacy Supports** • ***Interpretive language (i.e., making meaning of texts):*** Much of this exploration requires interpretation, where students analyze prompts within Desmos and make sense of new information. • ***Collaborative language (i.e., making meaning with others):*** Students perform both Desmos and Khan Academy individually but are highly encouraged to discuss issues they have within groups. Much of the exploration process requires making meaning with others in a group setting. • ***Productive language (i.e., communicating meaning with texts):***

	The Desmos activity includes prompts throughout that require students to justify their reasoning using written responses.
Eliciting Student Ideas and Lived Experiences ● Like above, I'll ask students to identify ratios in their own life and how understanding those ratios helped them in their goals. This will likely occur in individual conversations with students as I walk around the room, checking in on each student's progress in the activity. ● The Desmos activity includes multiple prompts asking students to describe their thinking using written responses. These responses are available to me in real-time as the students create them, and I can use those responses as a formative assessment, and to understand students' thought processes and progress throughout the activity.	**Talk and Translanguaging Moves** ● This Desmos activity has been translated and made available in eight languages, including English and Spanish. ● Students will be purposely grouped to have ELLs with bilinguals to aid in communication. Students can discuss with a partner in their native language. ● I'll elicit student thinking when they're stuck using phrases like, "What have you tried?" or "Where's your head at?" before pointing them on the right path. ● I'll probe students with phrases like "tell me more" when they initially express confusion. ● I'll revoice their thinking when applicable, and solicit alternative solutions from them or their group when solving problems together.
Beyond and Assess (*12:41 - 12:51, 10 minutes)*	
Synthesizing/Making Sense of Learning ● Near the end of class, I'll call the students back together and have them reflect on what they had learned. Since this Through activity was specifically meant for exploration, I'll encourage them to share their observations and use those to set their expectations for tomorrow's instruction. I'll instruct students to write those observations down in a question at the end of the Desmos activity, to be used in the next lesson. ● These observations and questions act as another kind of formative assessment, allowing me to tailor future learning.	**Language and Literacy Supports** ● ***Collaborative language (i.e., making meaning with others)*** Students will have the opportunity to share their observations and what they learned about rational functions. ● ***Interpretive language (i.e., making meaning of texts)*** Where possible, I'll encourage class discussion and opinions if any shared learnings can be further defined. ● ***Productive language (i.e., communicating meaning with texts)*** Students will be writing down their observations and justifying how they came to those observations.

Day #: 2 **# of minutes:** 62 minutes	**Brief Agenda:** ● Briefly review student learnings from the previous lesson ● Direct instruction on rational functions ● End with an example of all learnings shown using one exercise

Standard from your discipline:

- **A-APR.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. (Concept)
- **F-IF.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, showing end behavior. (Procedure)
- **F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Application)

Additional standard(s) from another discipline:

- None

Central concept/idea/performance: The central focus of this unit is to introduce rational functions and their properties, like asymptotes and end behavior, and build upon students previous knowledge of polynomials to solve real-world ratio-based problems. Standards A-APR.7 will direct students' conceptual understanding, while F-IF.7d and F-IF.4 will direct student learning of procedure and application, respectively.

Learning Objective(s): Students will be able to identify a rational function when written as an equation as well as when presented graphically, and identify properties of rational functions, like end behavior and vertical asymptotes. Students will be able to use this information to sketch a rough graph.

Into *(11:49 - 11:59 am - 10 minutes)*

Activating Prior Knowledge and Making Students Aware of the Central Focus ● We'll start this lesson by reviewing student learnings from the previous lesson. I'll reflect on what students have written, and use it as a jumping off point to transition to the Through activity of this lesson. Ideally, shared student feedback will help guide how I present the topics discussed in this lesson's Through activity, described below.	**Language and Literacy Supports** ● ***Interpretive language (i.e., making meaning of texts):*** This Into activity requires students to interpret and reinterpret the feedback they gave in the previous lesson. ● ***Collaborative language (i.e., making meaning with others):*** ● ***Productive language (i.e., communicating meaning with texts):***

Eliciting Student Ideas and Lived Experiences	**Talk and Translanguaging Moves**
• The Into part of this lesson will likely not include any elicitation of new ideas, primarily due to time constraints. If necessary I'll ask students to explain and/or expand upon their observations so those observations are more applicable to all students.	• Where possible, I'll redefine and revoice student input, expanding on it as necessary to help aid in student understanding. • I'll use cognates often, especially in reference to students' word choice.
Through *(11:59 - 12:41 - 42 minutes)*	
Engaging in Central Learning Activities and Texts	**Language and Literacy Supports**
• The Through activity of this lesson formally introduces rational functions to students, using mathematical definitions and terms. I will use an iPad connected to a projector to take notes in front of the whole class, using an application called Jamboard where students can follow along on the note taking on their own computers at their own pace. I'll cover all three standards listed above, presenting the concept (A-APR.7), describing procedure (F-IF.7d), and applying both using an example problem (F-IF.4). Specifically, this lesson will mathematically define a rational function, show how to find the domain of rational functions looking for removable discontinuities, examine end behavior, and identify vertical asymptotes. We'll also cover transformations of rational functions and relate their behavior to transformations of other functions that students had learned in prior lessons. • I'll include periods within the instruction for students to try example problems together in groups. Their results and feedback from tackling these problems acts as an informal formative assessment of the topics being presented.	• ***Interpretive language (i.e., making meaning of texts):*** Since this Through activity is primarily direct instruction, students will need to interpret mathematical concepts on screen. Where possible, I'll highlight key details, concepts and procedures using color, boldness, arrows and various organizational techniques. I'll also give students chances to make guesses and justify their thinking throughout. • ***Collaborative language (i.e., making meaning with others):*** I'll structure the instruction so students have time to apply what they have learned at each step and tackle example problems with their seating groups. • ***Productive language (i.e., communicating meaning with texts):*** After brief work time, I'll ask students to share their thought process and justify their result. Whether that result is correct or incorrect, I'll offer other students the opportunity to critique and provide a different interpretation and thought process.
Eliciting Student Ideas and Lived Experiences	**Talk and Translanguaging Moves**
• Throughout this note taking, I'll call on students to engage them in the lesson and encourage them to share their thought process. When	• Students will be purposely grouped to have ELLs with bilinguals to aid in communication. Students can

solving problems together as a class, I'll use a card system so that I can elicit students' understanding in a random order so as not to single anyone out. ● I'll provide opportunities for input at many steps along the way. Where we need to reference topics from previous lessons (say, when reviewing the leading term of a polynomial and its degree), I'll call on students to provide that information so that they can maintain active listening.	discuss with a partner in their native language. ● As above, I'll relate the word "rational" to its Latin root "rationalis", meaning "reasonable" or "logical", and describe how the Greeks specifically chose this word because of their mathematical views of that time period. I'll do the same for other words as well, like asymptote. ● I'll rely heavily on cognates when describing new topics. ● All notes I take will be available to students on their own computers so they can follow along at their own pace, including all terms and definitions.
Beyond and Assess (*12:41 - 12:51, 10 minutes)*	
Synthesizing/Making Sense of Learning ● I'll finalize this lesson with a single example problem that synthesizes all learnings into one. Students will be able to apply everything they've learned, gathering important information about a rational function to sketch a rough graph. Will do this together on screen, and I'll use the card-system to elicit students' thought processes. ● I will then remind them of the assigned homework for this week from their textbook.	**Language and Literacy Supports** ● ***Collaborative language (i.e., making meaning with others)*** ● ***Interpretive language (i.e., making meaning of texts)*** ● ***Productive language (i.e., communicating meaning with texts)*** This final activity relies heavily on productive language, where students have to justify their thought process while solving the problem together as a class. I'll probe students as necessary to refine their thinking and share it using mathematical terminology.

Day #: 3 **# of minutes:** 116 minutes	**Brief Agenda:** ● Warm up activity ● Homework review ● Light It Up! lab and worksheet ● End of class review

Standard from your discipline:

- **A-APR.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. (Concept)
- **F-IF.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, showing end behavior. (Procedure)
- **F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Application)

Additional standard(s) from another discipline:

- None

Central concept/idea/performance: The central focus of this unit is to introduce rational functions and their properties, like asymptotes and end behavior, and build upon students previous knowledge of polynomials to solve real-world ratio-based problems. Standards A-APR.7 will direct students' conceptual understanding, while F-IF.7d and F-IF.4 will direct student learning of procedure and application, respectively.

Learning Objective(s): Students will be able to identify a rational function when written as an equation as well as when presented graphically, and identify properties of rational functions, like end behavior and vertical asymptotes. Students will be able to use this information to sketch a rough graph.

Into *(10:37 - 10:52 am - 15 minutes)*

Activating Prior Knowledge and Making Students Aware of the Central Focus ● The warm up activity is presented on screen and shows four boxes, each containing an equation, picture, graph or some other mathematical concept. Students are asked to evaluate the content in each box and determine if that box is true or false. At least one box in the warm up will be considered false. Typically, these warm ups include both correct and incorrect mathematical concepts and ideas, as well as others that depend heavily on interpretation. This interpretation is a great launching point for discussion. Students will be asked to perform a think-pair-share, where they spend two	**Language and Literacy Supports** ● ***Interpretive language (i.e., making meaning of texts):*** Students will interpret the symbols, text and images on screen and will have to make meaning from it. They'll have time to interpret this on their own as well as with others. ● ***Collaborative language (i.e., making meaning with others):*** This warm up includes multiple modes of collaboration, discussing each concept with both their partner,

minutes thinking about the warm up themselves, two minutes with their team, and then two minutes walking around the room. At the end of the warm up, a team is chosen at random and they'll present their findings to the rest of the class.	team, and other teams within the class. Students will also use this time to persuade others about the truthiness of each of the boxes presented. ● ***Productive language (i.e., communicating meaning with texts):*** One team will present their findings at the end of the warm up. In doing so, they need to justify their answers and provide evidence why they think something is true or false.
Eliciting Student Ideas and Lived Experiences ● Students will work collaboratively with others during multiple portions of the warm up, communicating their ideas with others; and one team will have to present their ideas. ● After the presentation, I'll look for opportunities to elicit or refine the ideas presented, and probe for more information as necessary. ● I'll follow up by asking the class if anyone else had a different thought process or outcome, and use that as opportunity for discussion.	**Talk and Translanguaging Moves** ● Usually students present a limited amount so as to reduce their time "on stage". When possible, I'll have students "tell me more" after validating their ideas and contributions. ● Students are encouraged to present in the language they feel most comfortable. ● Students will be purposely grouped to have ELLs with bilinguals to aid in communication. Students can discuss with a partner in their native language.
Through *(10:52 - 12:23 - 91 minutes)*	
Engaging in Central Learning Activities and Texts ● This Through activity will give students a chance to apply knowledge about rational functions as well as explore properties of rational functions in real world scenarios. Students will be given a Light It Up! lab worksheet which provides multiple examples of applied problems related to rational functions. The last problem includes an actual lab activity using mirrors and laser pointers, where students shine laser pointers mirrors placed on the floor and then measure the height of the reflection on the wall. Through this activity, students will be able to gather data and learn that the relationship between the laser pointer's distance from the wall and the height of the laser pointer's reflection can be modeled as a rational function. They'll use this data to	**Language and Literacy Supports** ● ***Interpretive language (i.e., making meaning of texts):*** Students will have opportunity throughout the activity to record evidence, and express their reasoning orally, through writing as well as graphs and drawings. They'll work together in the activity and be able to critique the reasoning of others. ● ***Collaborative language (i.e., making meaning with others):*** This activity is a collaborative activity, and will require students to make meaning with others. I'll walk

create a rational function that they can then use to later predict outcomes of further tests.	around and check in with students as they're progressing, and help them understand the task as needed. ● ***Productive language (i.e., communicating meaning with texts):*** The lab worksheet includes multiple prompts where students must justify their process and conclusions in writing. I will encourage students to use "good math grammar" in their writing, and will periodically evaluate their answers to see if they are using proper mathematical language.
Eliciting Student Ideas and Lived Experiences ● This lab activity attempts to connect the mathematical work with a TV show students usually know about: The Simpsons. Each problem in the worksheet is within The Simpsons theme using ideas and characters from the TV show. Using this worksheet for the activity was intentional, as it will provide existing avenues for students to access new learnings.	**Talk and Translanguaging Moves** ● Students will be purposely grouped to have ELLs with bilinguals to aid in communication. Students can discuss with a partner in their native language. ● I'll read and summarize the first page of the activity so students can understand their task via both visual and auditory sensory input. ● When students are stuck, I'll give them a chance to provide their thinking before attempting to correct or direct it. This means that I'll respond to questions and statements like "I don't understand this" or "what does this mean?" with "Well, tell me about your thought process so far."
Beyond and Assess (*12:41 - 12:51, 10 minutes)*	
Synthesizing/Making Sense of Learning ● At the end of the class, I'll bring everyone back together to discuss their thoughts about that activity. I'll ask students to tell me about their progress and how far along they were, as well as asking them about any confusions or misunderstandings. I'll then pull up the questions and observations they created in Lesson 1, reviewed in Lesson 2, and are now coming back to in Lesson 3. I'll end the discussion by having them reflect on how their	**Language and Literacy Supports** ● ***Collaborative language (i.e., making meaning with others)*** This Through activity is collaborative in the sense that we will be having a full class discussion. I'll attempt to facilitate a student-to-student discussion rather than a student-teacher-student discussion, if possible.

observations and thinking evolved.	• ***Interpretive language (i.e., making meaning of texts)*** Students will be able to share their thinking, and give evidence and reasoning for why their thinking may have changed. They'll also be able to share what they're still struggling with, as well as critique others' reasoning as the discussion progresses. • ***Productive language (i.e., communicating meaning with texts)*** Here, students will talk about their experience over the course of all the lessons and reflect on how their thinking has changed.

Day #: 4 **# of minutes:** 62 minutes	**Brief Agenda:** ● Brief review of rational functions ● Assessment

Standard from your discipline:

- **A-APR.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. (Concept)
- **F-IF.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, showing end behavior. (Procedure)
- **F-IF.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Application)

Additional standard(s) from another discipline:

- None

Central concept/idea/performance: The central focus of this unit is to introduce rational functions and their properties, like asymptotes and end behavior, and build upon students previous knowledge of polynomials to solve real-world ratio-based problems. Standards A-APR.7 will direct students' conceptual understanding, while F-IF.7d and F-IF.4 will direct student learning of procedure and application, respectively.

Learning Objective(s): Students will be able to identify a rational function when written as an equation as well as when presented graphically, and begin to identify properties of rational functions, like end behavior and vertical asymptotes.

Into *(11:49 - 11:59 am - 10 minutes)*

Activating Prior Knowledge and Making Students Aware of the Central Focus	**Language and Literacy Supports**
● At the beginning of this lesson, I'll do a brief review of rational functions before directing students to perform the assessment. I'll also offer the opportunity to answer any questions before the assessment is started. All this will be very brief, as the rest of the time will be required for the assessment.	● ***Interpretive language (i.e., making meaning of texts):*** There won't be much opportunity for interpretation beyond the few minutes I spend reviewing and answering questions, given that this lesson's primary goal is the assessment. ● ***Collaborative language (i.e., making meaning with others):*** Same as above. ● ***Productive language (i.e., communicating meaning with texts):***

	Same as above.
Eliciting Student Ideas and Lived Experiences ● There will be very little, as the goal of this lesson is to perform the assessment.	**Talk and Translanguaging Moves** ● I will reiterate and describe necessary topics and terminology where possible. This review and question answering will take only a few minutes at most. I'll use cognates during explanations as the opportunity arises.
Through *(11:59 - 12:51 - 52 minutes)*	
Engaging in Central Learning Activities and Texts ● This assessment will consist of three sections, each targeting one of the standards listed above (A-APR.7: Concept, F-IF.7d: Procedure, and F-IF.4: Application). One question will require students to analyze and reflect on rational functions and their properties, and will require a written response where they justify their knowledge of the concept (Concept). The second question will give students a rational function and ask them to identify zeros, removable discontinuities, asymptotes and end behavior, and use that information to sketch a rough graph (Procedure). The third will present students with a word problem to analyze and have them make conclusions based on their analysis using rational functions (Application).	**Language and Literacy Supports** ● ***Interpretive language (i.e., making meaning of texts):*** There are very little interpretive supports here since this is the assessment, and ideally all those supports would have occurred in previous lessons. ● ***Collaborative language (i.e., making meaning with others):*** Since this is an assessment, there will be no collaboration. ● ***Productive language (i.e., communicating meaning with texts):*** Students will need to justify their answers in writing, explain their procedures, and draw graphs in order to answer the problems presented to them. For each section of the assessment, the associated standard and its text will be displayed on the assessment so students know exactly what work they are expected to perform.
Eliciting Student Ideas and Lived Experiences ● The assessment, as described above, is explicitly tailored to eliciting students' ideas. The questions are built such that students will have to show master of the concept, procedure and application.	**Talk and Translanguaging Moves** ● Not much in this section, since the focus is the assessment.

Beyond and Assess (*12:51 - 12:51, 0 planned minutes)*	
Synthesizing/Making Sense of Learning ● There is no official Beyond activity for this lesson, as its primary focus is on performing the assessment. Students have up until the bell rings to finish. ● I will assign weekend reading for all students during this time, and those that finish the assessment early can begin working on it. This weekend reading will talk about the history of rational functions and how they're useful in the real world.	**Language and Literacy Supports** ● ***Collaborative language (i.e., making meaning with others)*** None in this section. ● ***Interpretive language (i.e., making meaning of texts)*** The weekend reading is heavily interpretive, and doesn't require students to "do math". Instead, it informs students about the history of the mathematical topics we learned, allowing them to build even greater conceptual understanding. ● ***Productive language (i.e., communicating meaning with texts)*** None in this section.